ISTS-2017-d-047／ISSFD-2017-047

Utilisation of the "Dzhanibekov's Effect" for the Possible Future Space Missions

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(Received April 24th, 2017)

This paper is uncovering the mystery of the interesting "Dzhanibekov's effect" (often also called Dzhanibekov's phenomenon), providing systematic detailed explanation of the intriguing phenomenon using the numerical simulation methods and tools, employing non-linear equations of motion of the rigid bodies. Based on the developed simulation model, we also explore the possibilities of utilisation of the "Dzhanibekov's Effect" for possible future new space missions, employing periodic change in the attitude orientation of the spacecraft. In our conceptual designs, in particular, we consider novel cases of the control of the dynamics of the spinning rotating spacecraft via active change of its inertial properties. This, for example, enables for the spacecraft with initial stable axial spin, at the desired time, to be transferred into the "Dzhanibekov's" unstable flipping mode (maybe, for changing its head attitude by 180° or observations) and then, if needed, to return back to the initial stable spin. Essentially, we present the method of controlled switching ON and OFF of the unstable periodic flipping motion of the spacecraft (known as "Dzhanibekov's effect") via controlled morphing of the spacecraft. This paper also presents the 6-masses conceptual design of the spacecraft, capable of producing required morphing, necessary for activation or de-activation of the flipping tumbling, without use of the gyroscopes. We also produce geometric interpretation of the "flip ON/OFF" developed method.

Key Words: "Dzhanibekov's effect", Rigid body dynamics, Euler's equations of motion, Morphing spacecraft, Tumbling motion

Nomenclature

ψ, θ, ϕ	:	Euler angles
ω_x, ω_y, ω_z	:	components of the angular velocity
d	:	derivative
$\mathcal{G}$	:	centre of the mass of the rigid body
$\vec{\mathbf{H}}(t)$	:	angular momentum vector
I_{xx}, I_{yy}, I_{zz}	:	principal moments of inertia
m_x, m_y, m_z	:	dumbbell masses in the 6-mass spacecraft
$\mathcal{M}$	:	mass matrix
Maa, Mbb, Mcc	:	moments of inertia in original Euler's work
N_x, N_y, N_z	:	torque components
$\mathcal{P}$	:	pivot point
P, Q, R	:	torque components in original Euler's work
r_x, r_y, r_z	:	axial positions of the spacecraft masses
t	:	time
x, y, z	:	principal axes of the rigid body
$\mathbf{x}$	:	system's states
Subscripts		
f	:	final
i	:	initial

1. Introduction

1.1. Discovery of the phenomenon in space

Vladimir Aleksandrovich Dzhanibekov is a famous Russian cosmonaut (shown in Fig. 1), who with his five space flights is recognised the champion in this category. In fact, he has spent in space 145 days, 15 hours and 35 seconds.[2] Time duration of his open space walks is 8 hours and 34 minutes.

His first flight was in 1978. And during his fifth space flight, on 25-June-1985, he worked on-board of the space assembly "Salyut-7"-"Soyuz T-13", unpacking the payloads, delivered from the Earth by "Progress-24 (#125)" supply transport vehicle. All arrived payloads, sent to the orbital space station "Salyut-7", were constrained to prevent their movements during the launch and flight of the space vehicle. The fixation of the payloads to the bases was secured by the classical fixation elements, involving long threaded rods and fixation "ear wing butterfly" nuts. The unpacking process involved exhausting unscrewing of many nuts, which required for each of the nuts to be rotated many times for them to travel significant distances along their corresponding fixing rods. To speed up the process, Vladimir Dzhanibekov has applied a significant torque impulse to the wing of the nut, which resulted in the initiation of the fast rotation of the nut and its conjugated translational motion along the threaded rod. The impulse was sufficient for the nut to complete unscrewing process on it own, and then to leave the rod. From this moment the nut continued its free flight, travelling along the axis of the left rod, while still

Fig. 1. Vladimir Aleksandrovich Dzhanibekov[1]

being in rotation about this axis. After travelling the translational distance about 42 cm, the nut, after its apparent stable and undisturbed flight, suddenly changed its axial orientation by 180 degrees, simultaneously changing its direction of rotating to opposite in the body-axis coordinate system and continued its flight backwards. It was even more amazing for the discoverer to realize, that this pattern of motion has been repeated in the periodic sequence, without any apparent external force applied. Using gymnastics terminology, it looked almost like the nut was performing the “Roundoff Backflip”. This spectacular behaviour in the weightless environment of “flipping” of the rigid body on the axis of main rotation, later was named the “Dzhanibekov’s effect” or “Dzhanibekov’s phenomenon”. Attracting attention of scientists and engineers, this discovery has even prompted a new hypothesis, that the Earth, similar to the “wing nut” in Dzhanibekov’s phenomenon, is regularly performing its flips, but with a period of approximately 12,000 years.[3] In view of its potential importance, as a matter of precaution, the Dzhanibekov’s discovery was classified by authorities for 10 years.[4]

Later on, the Dzhanibekov’s phenomenon, which initially was perceived by some as counter-intuitive or even mysterious, has been explained in various journal and on-line publications:[5–7] the Euler’s equations have paved the theoretical ground to its scientific manifestation. Various popular videos and demonstrations became available to the wide audience. On numerous occasions, Vladimir A. Dzhanibekov himself explained his discovery in various lectures, TV programs and interviews.[4,8]

1.2. Demonstrations of the Dzhanibekov’s phenomenon on board of the International Space Station

Later on, the Dzhanibekov’s phenomenon has been reproduced and observed during numerous demonstrations on board of International Space Station. Interested readers are referred to multiple videos in the media.

A series of experiments with various rigid bodies, including cylinders, cubes and right rectangular prisms was conducted on board of the ISS by Dan Burbank and Anton Shkaplerov, members of the 30-th expedition.[9]

Japanese astronaut Koichi Wakata (JAXA), has also conducted an experiment on board of ISS with spinning and tumbling pliers.[10]

Another similar video is where astronaut Kevin Ford (NASA) is conducting another experiment on board of ISS (34-th expedition) with spinning and tumbling pliers.[11]

Richard Garriott, pioneer in commercial space travel, has also run a series of outreach program experiments on board of the ISS and in the video[12] demonstrates Dzhanibelkov’s effect, using a deck of playing cards.

Another video on the topic shows a tumbling T-handle[13] experiment on board of the ISS and is a wonderful illustration of the instability of rotation about an asymmetric object’s intermediate principal axis.

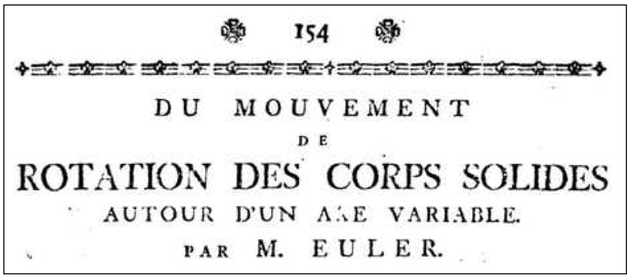
The Dzhanibekov’s phenomenon, and also so called “tennis racket phenomenon” were explained using Euler’s equations for an unconstrained rigid body.[5] It has been realised that rotation of the body about the axis with intermediate principal moment of inertia becomes unstable, resulting in sudden change of its attitude.

1.3. Historical perspectives: Euler’s equations

Leonhard Euler (April 15, 1707 - Sept. 18, 1783) was a famous Swiss physicist and mathematician (the most eminent of the 18th century and one of the greatest in history), who made key contributions to various fields of mathematics and mechanics, leaving long-lasting heritage of more than 500 books and papers. It has been computed that his publications during his working life averaged about 800 pages a year. His portrait is presented in Fig. 2. Among numerous Euler’s works, where

Fig. 2. L.Euler’s portrait from the University of Tartu collection.[14]

he developed rigid-body dynamics, very influential publication 15) has a very special place in history. It presented Euler’s equations for the dynamics of a rigid body, widely used in modern engineering and science. In Fig. 3 we show the title of the publication, available from the Euler’s archive[16] and the reproduced famous Euler’s equations, exactly as they appeared in the original work.[15] In the equations in Fig. 3, M, accordingly to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DU MOUVEMENT
DE
ROTATION DES CORPS SOLIDES
AUTOUR D'UN AXE VARIABLE.
PAR M. EULER.

(a) The title of the historic L.Euler’s work 15), dated by 1758.

$$P = Maa.\ \frac{d.\upsilon \cos\alpha}{2g\,dt} + M(cc - bb).\ \frac{\upsilon\upsilon \cos\beta \cos\gamma}{2g}$$
$$Q = Mbb.\ \frac{d.\upsilon \cos\beta}{2g\,dt} + M(aa - cc).\ \frac{\upsilon\upsilon \cos\gamma \cos\alpha}{2g}$$
$$R = Mcc.\ \frac{d.\upsilon \cos\gamma}{2g\,dt} + M(bb - aa).\ \frac{\upsilon\upsilon \cos\alpha \cos\beta}{2g}$$

(b) Euler’s equations as they appeared in the original L.Euler’s work 15).

Fig. 3. Famous Euler’s equations for the rigid body dynamics.[16]

Euler, is the weight, and Euler emphasised that Maa, Mbb and Mcc are the inertia moments of the body along the three fixed axes[17] and P, Q and R are the moments of the forces along the principal axes.

In modern language, the Euler's equations in Fig. 3(a) can be written as follows:

$$\begin{aligned} \sum N_x &= I_{xx}\ \dot{\omega}_x + (I_{zz} - I_{yy})\ \omega_y\omega_z \\ \sum N_y &= I_{yy}\ \dot{\omega}_y + (I_{xx} - I_{zz})\ \omega_z\omega_x \\ \sum N_z &= I_{zz}\ \dot{\omega}_z + (I_{yy} - I_{xx})\ \omega_x\omega_y \end{aligned} \tag{1}$$

where x, y, z are the principal axes of inertia fixed to the body; the components of angular velocity in this system are $\omega = (\omega_x, \omega_y, \omega_z)$, the torque is $N = (N_x, N_y, N_z)$ and the diagonal elements of the inertia tensor are I_{xx}, I_{yy} and I_{zz}.

The equations, known as "Euler's equations" for a rigid body, referred to as principal inertia axes, and with the angular velocity components in terms of the angles α, β, γ, which are the angles subtended by the rotation axes with the principal ones fixed in the body. It could be said that these are the Euler angles, although actually they are usually defined by applying the rotation operator to the axes fixed on the body, so that each angle is related to the angular velocities of rotation known as precession, nutation and spin.

2. Numerical Simulation of the "Dzhanibekov's Effect"

2.1. Equations of motion

Euler's equations .(1), in the general case, can be applied for moments summed about any point $\mathcal{P}$, where $\mathcal{P}$ is a point on the rigid body that is attached to a fixed pivot in the inertial reference system. However, in this case the inertia properties should be calculated relative to the point $\mathcal{P}$.

In our study we will apply the Euler's equations for moments summed about the center of mass $\mathcal{G}$ of the rigid body, free from any external torques ($N_x = N_y = N_z = 0$) and in the further notations we will imply that I_{xx}, I_{yy}, I_{zz} are principal moments of inertia of the body with respect to the $\mathcal{G}$:

$$\begin{aligned} I_{xx}\,\dot{\omega}_x - (I_{yy} - I_{zz})\ \omega_y\omega_z &= 0 \\ I_{yy}\,\dot{\omega}_y - (I_{zz} - I_{xx})\ \omega_z\omega_x &= 0 \\ I_{zz}\,\dot{\omega}_z - (I_{xx} - I_{yy})\ \omega_x\omega_y &= 0 \end{aligned} \tag{2}$$

The matrix form of the above is:

$$\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{Bmatrix} = \begin{Bmatrix} (I_{yy} - I_{zz})\ \omega_y\omega_z \\ (I_{zz} - I_{xx})\ \omega_z\omega_x \\ (I_{xx} - I_{yy})\ \omega_x\omega_y \end{Bmatrix} \tag{3}$$

In order to be able to describe instantaneous orientation of a rigid body with respect to a fixed coordinate system, we will use the angles ψ, θ and ϕ, the Euler angles[18] :

$$\begin{aligned} \omega_x &= \dot{\psi}\ \sin\theta\ \sin\phi + \dot{\theta}\ \cos\phi \\ \omega_y &= \dot{\psi}\ \sin\theta\ \cos\phi - \dot{\theta}\ \sin\phi \\ \omega_z &= \dot{\psi}\ \cos\theta + \dot{\phi} \end{aligned} \tag{4}$$

which can also be written in the matrix form:

$$\begin{bmatrix} \sin\theta\,\sin\phi & \cos\phi & 0 \\ \sin\theta\,\cos\phi & -\sin\phi & 0 \\ \cos\theta & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{Bmatrix} = \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \tag{5}$$

For solving the rigid body dynamics problems, using numerical methods, we combine matrix equations (3) and (5) into a single equation:

$$\begin{bmatrix} I_{xx} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{yy} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin\theta\,\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & \sin\theta\,\cos\phi & -\sin\phi & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{Bmatrix} = \begin{Bmatrix} (I_{yy} - I_{zz})\ \omega_y\omega_z \\ (I_{zz} - I_{xx})\ \omega_z\omega_x \\ (I_{xx} - I_{yy})\ \omega_x\omega_y \\ \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \tag{6}$$

2.2. Programming considerations

Ordinary differential equations can be efficiently solved using Runge-Kutta methods. MATLAB® has a set of specialised procedures, including `ode45`, `ode23`, `ode113`, `ode15s`, `ode23s`, `ode23t`, `ode23tb`, `ode15i`, to deal with various tasks, for example, described by the ordinary differential equation in the classical form: $\{\dot{\mathbf{x}}\} = \{f(t, \mathbf{x})\}$.

There is also a very useful option enabling solution of the problems, involving so called "mass" matrix $\mathcal{M}$:

$$[\mathcal{M}(t, \mathbf{x})]\{\dot{\mathbf{x}}\} = \{f(t, \mathbf{x})\} \tag{7}$$

This option, accessible via the `odeset`, in some cases can improve efficiency and can also handle cases when the mass matrix is singular (non-invertible). As it can be seen, our Eqs. (6) correspond to the format given with Eq. (7), therefore, we use MATLAB® `ode` procedure in conjunction with the "mass matrix" option to simulate dynamic behaviour of the morphing spacecraft models.

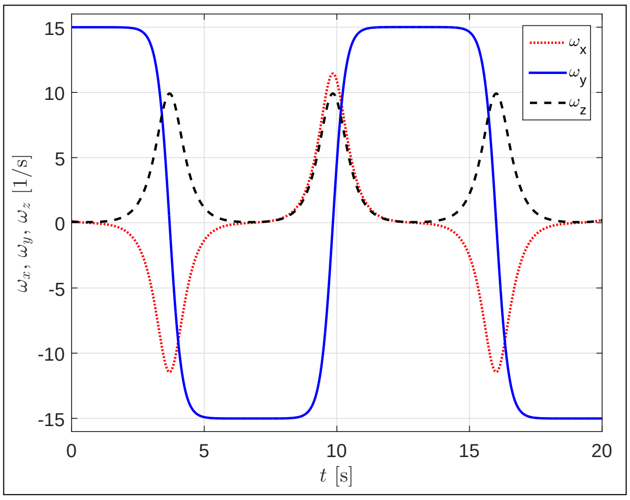
2.3. Study Case-1: results

Let us consider a task of simulating the motion of the rigid body with the following parameters: $I_{xx} = 0.3$, $I_{yy} = 0.35$, $I_{zz} = 0.4$ (all in kg*m^2), with the initial conditions ${}_i\omega_x = 0.1$, ${}_i\omega_y = 15$, ${}_i\omega_z = 0.1$ (all in rad/s). Equations (6) were solved numerically and main results are given in Fig. 4. Their observation confirms periodical flipping of the system: indeed, ω_x in Fig. 4.(a) is periodically changing its sign. Fig. 4(b) confirms that during the "flipping" motion, the angular momentum in the system is concerved. At last, Fig. 4(c) shows that while ψ is monotonically increasing, the ϕ pattern is quite different: there are evident "plateau" segments corresponding to small changes in ϕ around 0°, 180°, 360°, etc. However, the most important observation in the context of this paper is presence of the multiple zero-crossings for various components of the angular velocity, in particular, for ω_x and ω_y in the test case.

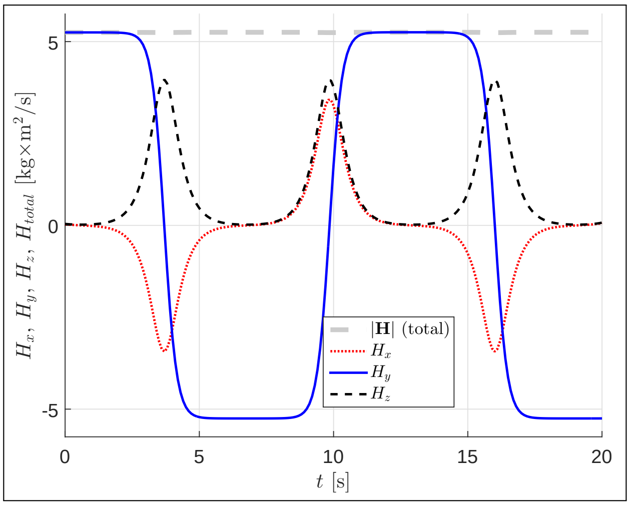
3. Proposing New Spacecraft Designs/Missions, Utilising Dzhanibelov's Phenomenon

3.1. Proposing Idea of "Switching ON/OFF" Dzhanibekov's spacecraft flipping by controlled morphing of the tumbling object

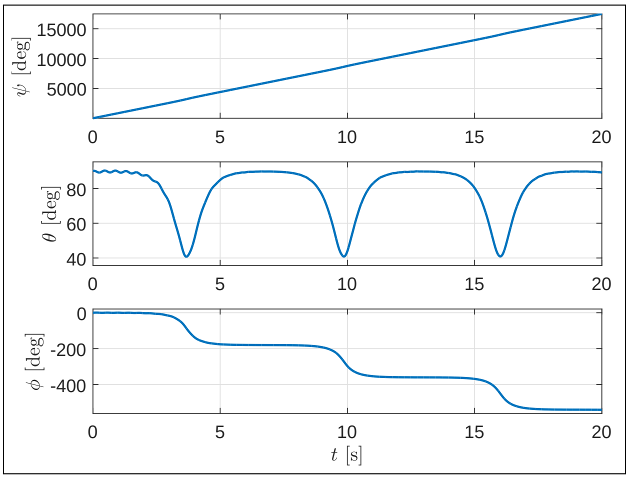
Flipping motion of the rigid body, during which the direction of the angular velocity of the main rotation, let say, ω_y, is



(a) Time histories of the angular velocity components of the spacecraft.



(b) Time histories of the angular momenta of the spacecraft.



(c) Time histories of the angular momenta of the spacecraft.

Fig. 4. Dzhanibekov's effect: simulation results for the study Case-1.

intermnittenly changing to opposite, is called "Dzhanibekov's effect". It is a consequence of the momet of inertia, associated with the main rotation, being between two other values of the moments of inertia, I_{xx} and I_{zz}, in other words, having an intermediate value among principal moments of inertia.

What if there is a need to stop or suspend for some time the flipping unstable motion of the object?

To solve this task, we are proposing to utilise the controllable morphing of the rigid body (the spacecraft, in the context of this paper). We are proposing for the purpose of stabilisation of the object to purposely change the mass distribution within the spacecraft, after which the intermediate moment of inertia becomes the smallest *or* largest among all principal moments of inertia. In the illustration case, where we selected y axis to be the axis of the main rotation, the condition for the unstable "Dzhanibekov's effect"-type motion can be written as:

$$I_{xx} < I_{yy} < I_{zz}. \tag{8}$$

However, if via special design of the spacecraft, enabling the change of its principal moments of inertia (via mechanical or other means), the targeted value of I_{yy} is in controllable way forcefully "moved" outside the embrace of I_{xx} and I_{zz}, then the condition of instability Eq. (8) would no longer be satisfied and the unstable motion would be "switched OFF"!

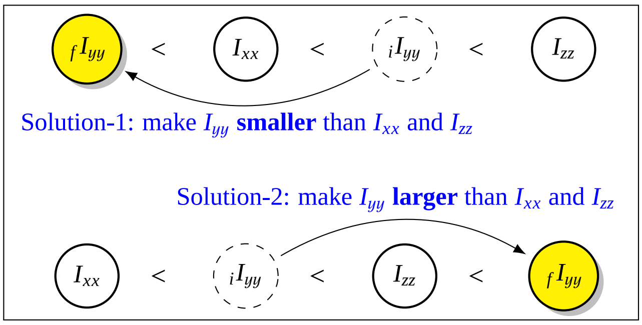


Fig. 5. Possible conceptual solutions for stabilising an unstable spacecraft with its main rotation about y axis.

Conceptually, this proposition can be illustrated with the diagram in Fig. 5, which presents *two* solutions. The first conceptual solution involves reduction of initial value of I_{yy} (which we denote as ${}_iI_{yy}$) to its new (or final) value ${}_fI_{yy}$, being smaller than I_{xx} value. And the second solution involves increase of the initial value of I_{yy} (which we denote as ${}_iI_{yy}$) to its new value ${}_fI_{yy}$, being larger than I_{zz}.

For the numerical verification of the concept, let us assume the following demonstration values: $I_{xx} = 0.3$, $I_{yy} = 0.35$ and $I_{zz} = 0.4$ (all in kg*m^2), which are conforming with the general condition Eq.(8) of the flipping unstable motion, which would result if the main rotation about y axis is initiated. And in this case, in order to test the concept of "switching OFF" the flipping motion, we will change ${}_iI_{yy} = 0.35$ to its new value of ${}_fI_{yy} = 0.2$ (solution-1) or ${}_fI_{yy} = 0.5$ (solution-2).

However, in order to proceed with the numerical simulations, we need to expand the Euler equations, allowing variations in the moments of inertia of the rigid body.

3.2. Equations of motion: extending Euler's equations

In order to simulate the cases of the morphing spacecraft with variable moments of inertia, we need to extend classic Euler's Eqs. (1). We note that the sum of the moments about the center of mass of a rigid body due to external forces and couples equals to the rate of change of the angular momentum about the center of mass:[18]

$$\sum \vec{\mathbf{N}} = \left.\frac{d\vec{\mathbf{H}}}{dt}\right|_{Inertial} = \left.\frac{d\vec{\mathbf{H}}}{dt}\right|_{Body} + \vec{\omega} \times \vec{\mathbf{H}}$$

Also, the components of the angular momentum vector, $\vec{\mathbf{H}}(t)$, with respect to the body-axis frame can be expressed by the product between the principal moment of inertia matrix $I_{\mathcal{G}}$ and the components of the angular velocity vectors as follows:

$$\vec{\mathbf{H}}(t) = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}$$

Therefore, extended Euler's equations can now be written as:

$$\begin{bmatrix} \dot{I}_{xx} & 0 & 0 \\ 0 & \dot{I}_{yy} & 0 \\ 0 & 0 & \dot{I}_{zz} \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} + \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{Bmatrix} +$$
$$\begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

For solving the *morphing* rigid body dynamics problems, using numerical methods, we combine matrix Eqs. (9) and (5) into a single equation:

$$\begin{bmatrix} I_{xx} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{yy} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin\theta\,\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & \sin\theta\,\cos\phi & -\sin\phi & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{Bmatrix} =$$
$$\begin{Bmatrix} (I_{yy} - I_{zz})\ \omega_y\omega_z - \dot{I}_{xx}\ \omega_x \\ (I_{zz} - I_{xx})\ \omega_z\omega_x - \dot{I}_{yy}\ \omega_y \\ (I_{xx} - I_{yy})\ \omega_x\omega_y - \dot{I}_{zz}\ \omega_z \\ \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \tag{10}$$

Equations (10) are the main equations, used in this paper and solved using `ode` MATLAB® Runge-Kutta solver, with "mass matrix" option, as per Eq. (7).

4. Proposing Geometric Morphing of spacecraft: Conceptual Considerations

To demonstrate the feasibility of the controllable behaviour of the spacecraft, let us consider a simple conceptual model of the morphing spacecraft, constructed as an axisymmetric set of three orthogonal dumbbells, each of which has negligible mass of the rod, connecting two equal concentrated masses at its ends. Let us also assume, for conceptual simplicity, that three dumbbells are connected at the middle points of their rods, and the corresponding masses m_x, m_y and m_z are located at the distances r_x, r_y and r_z from the axes of rotation x, y and z, as shown in Fig.(6). In the illustrated conceptual design, morphing of the spacecraft is achieved via independent synchronised control of the position coordinates $r_x = r_x(t)$, $r_y = r_y(t)$ and $r_z = r_z(t)$ of the masses m_x, m_y and m_z.

The principal moments of inertia of the system can be calculated as follows:

$$\begin{aligned} I_{xx} &= 2m_y r_y^2 + 2m_z r_z^2 \\ I_{yy} &= 2m_z r_z^2 + 2m_x r_x^2 \\ I_{zz} &= 2m_x r_x^2 + 2m_y r_y^2 \end{aligned} \tag{11}$$

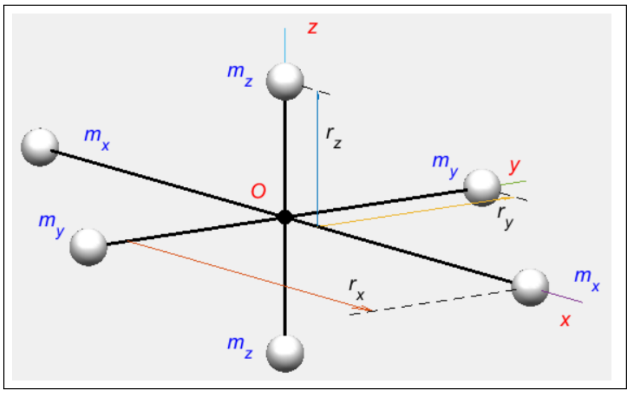
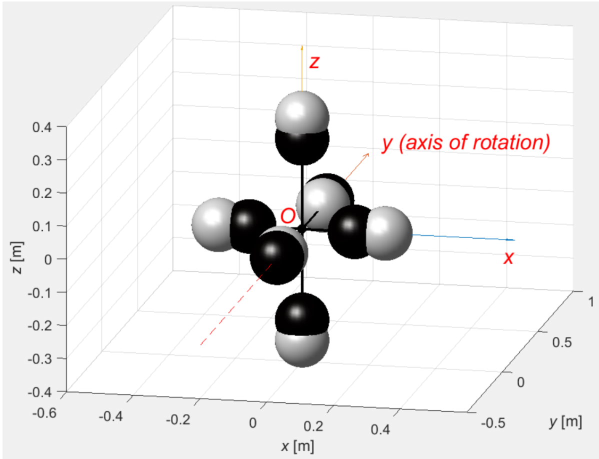
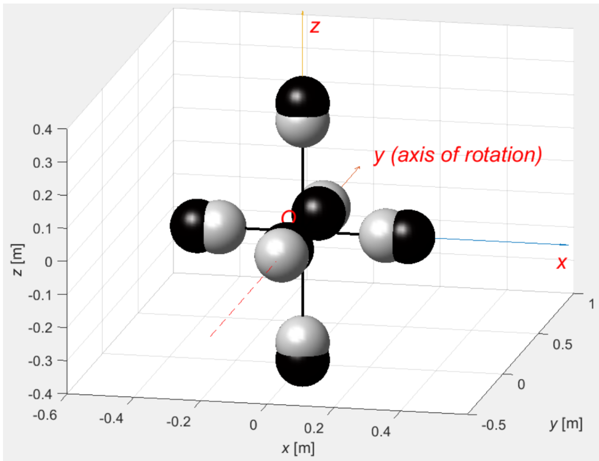


Fig. 6. Six-masses conceptual model of the morphing spacecraft.



(a) Solution-1, as per Fig. 5.



(b) Solution-2, as per Fig. 5.

Fig. 7. Conceptual 6-masses design of the morphing spacecraft, capable of self-transfer from unstable "Dzhanibekov's effect"-type flipping motion to stable motion (and vice versa): white spheres - unstable configuration for y main rotation, black spheres - stable configuration.

Then by adding all equations in (11), we can get:

$$\frac{1}{2}\left(I_{xx} + I_{yy} + I_{zz}\right) = 2\left(m_x r_x^2 + m_y r_y^2 + m_z r_z^2\right) \tag{12}$$

Then, subtracting from (12) consecutively each of Eq. (11), we can get:

$$r_x = \sqrt{\frac{I_{yy} + I_{zz} - I_{xx}}{2\,m_x}}$$
$$r_y = \sqrt{\frac{I_{zz} + I_{xx} - I_{yy}}{2\,m_y}} \tag{13}$$
$$r_z = \sqrt{\frac{I_{xx} + I_{yy} - I_{zz}}{2\,m_z}}$$

Let us assume, for the illustration purpose, that $m_x = m_y = m_z = 1$kg, $I_{xx} = 0.3$kg*m^2, $I_{xx} = 0.35$kg*m^2, $I_{xx} = 0.4$kg*m^2. Then, for the case of the tumbling spacecraft considered in Section **2.3.**, we can find the initial radial positions of the spacecraft masses, using Eqs.(13):

$$_ir_x = 0.2500 \text{ m}, \quad _ir_y = 0.2958 \text{ m}, \quad _ir_z = 0.3354 \text{ m} \tag{14}$$

These values for the unit masses would ensure, that the inertial properties of the spacecraft are $I_{xx} = 0.3$kg*m^2, $I_{yy} = 0.35$kg*m^2, $I_{zz} = 0.4$kg*m^2. Note that in our example here I_{yy} has an *intermediate* value among all principal moments of inertia: $I_{xx} < I_{yy} < I_{zz}$, therefore if the spacecraft is provided with the initial angular velocities $\omega_x = 0.1$rad/s, $\omega_y = 15$rad/s and $\omega_z = 0.1$rad/s, with the prevailing rotation about y body axis, then the spacecraft rotation about this axis would be unstable and classical “Dzhanibekov’s effect” periodic flipping would be observed.

It will be shown in Section **5.2.**, that if during the ”flipping” motion, at the instant, when the angular velocities $\omega_x = \omega_z$ are close to zeros, the moment of inertia $_iI_{yy} = 0.35$kg*m^2 is rapidly changed to its new value of $_fI_{yy} = 0.2$kg*m^2, then the nature of the followed motion of the system would change from unstable “flipping” to stable. We call it “switching OFF” the flipping motion.

This will be occurring because the moment of inertia I_{yy} stops being the intermediate value and the rotation about y body axis is becoming stable, without changes in the direction of ω_y.

The new values of the position radii, corresponding to the “solution-1” in Fig. 5, can be calculated using Eqs.(13):

$$_fr_x = 0.1581 \text{ m}, \quad _fr_y = 0.3536 \text{ m}, \quad _fr_z = 0.2739 \text{ m} \tag{15}$$

The spacecraft masses at these radius positions are shown in Fig.7(a) with dark color.

The flipping motion can be also stopped, using “solution-2”, shown in conceptual Fig. 5. For the purpose of the illustration of the concept, let us consider rapid increase of the I_{yy} from its initial value of $_iI_{yy} = 0.35$kg*m^2 to its new value of $_fI_{yy} = 0.5$kg*m^2. The new values of the position radii, corresponding to the “solution-2” in Fig. 5, can be calculated using Eqs.(13):

$$_fr_x = 0.2500 \text{ m}, \quad _fr_y = 0.2958 \text{ m}, \quad _fr_z = 0.3354 \text{ m} \tag{16}$$

The spacecraft masses at these radius positions are shown in Fig. 7(b) with dark color.

The morphing of the spacecraft from the initially unstable configuration [as per Eq. (14)], associated with the ”flipping” motion, to its final stable configuration [as per Eq. (15) or (16) and Solution-1 or 2 in Fig. 5], are shown in Fig. 7, where masses for the initial configuration are shown in white, whereas the masses for the final configuration are shown in black color.

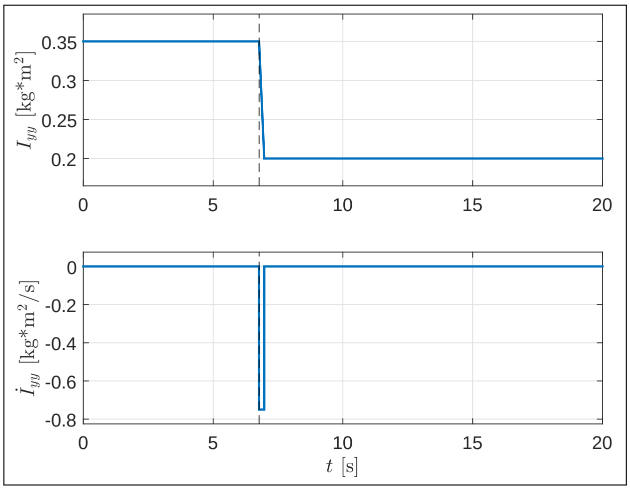


Fig. 8. Time history of the controlled manipulation with the moment of inertia I_{yy} to stop flipping motion of the system (Case-2).

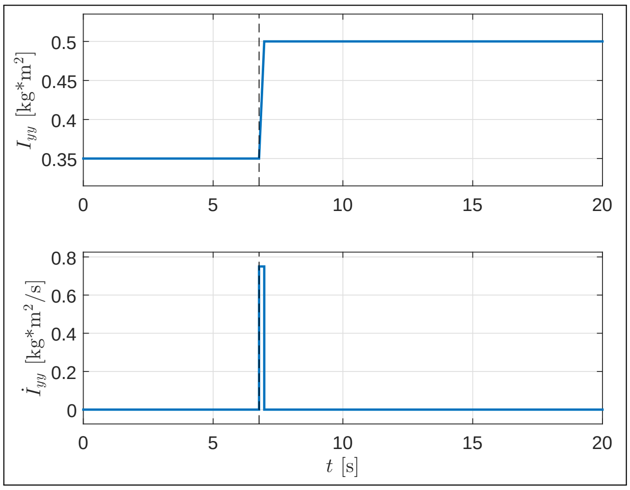
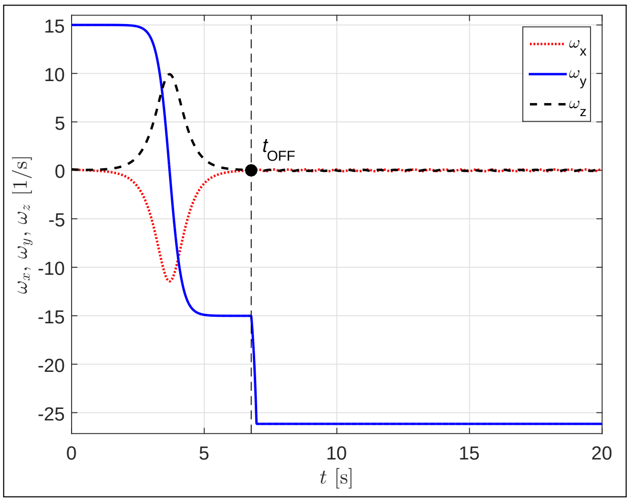


Fig. 9. Time history of the controlled manipulation with the moment of inertia I_{yy} to stop flipping motion of the system (Case-3).

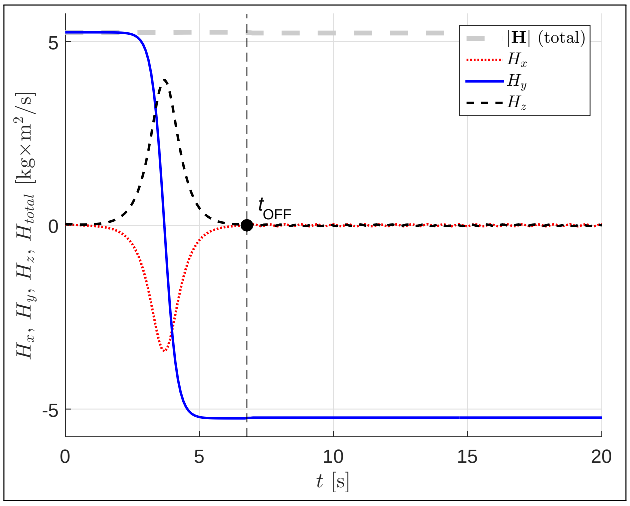
5. Motion of Spacecraft with Geometric Morphing

5.1. Study Case-2: ”Switching OFF” flipping motion of the spacecraft after one flip (solution-1)

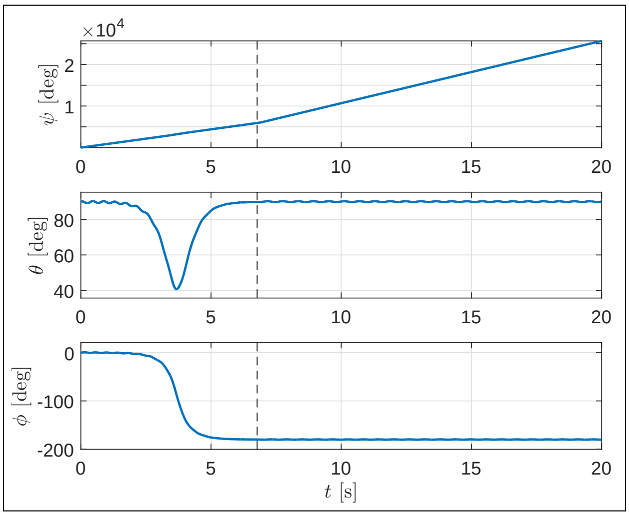
Figure 4 shows that at the instant $t = 6.77$ s, the angular velocity ω_y has its highest value and ω_x changes its value from negative to positive. It is believed that this instant, corresponding to the most prominent rotation about y-body axis, would be the best time to apply morphing to the spacecraft. In our demo case the moment of inertia I_{yy} is changed from 0.35 to 0.2, as per Fig. 8 within relatively short period of time of 0.2 s. Results of the simulation are given with Fig. 10. Figure 10(a) shows that the simulated morphing led to the step-type increase of the angular velocity ω_2 of the body and did not initiate significant oscillations in ω_x and ω_z. In contrast to Case-1, where ω and H plots had similar shapes, in the Case-2 these plots are different. Figure 10(b) shows that morphing did not change the angular momentum H_y and after the morphing was completed, the value of H_y stayed almost unchanged, evidencing that attempt to stop the “flipping” motion was successful. At last, note that as the stabilised value of $\phi = 180°$, the stabilised spacecraft is flying *backwards*, with its initial heading attitude changed by 180°!



(a) Time histories of the angular velocity components of the spacecraft.



(b) Time histories of the angular momenta of the spacecraft.



(c) Time histories of the angular momenta of the spacecraft.

Fig. 10. Simulation results for the study Case-2: switching off the "flipping" motion of the spacecraft. *Note:* t_{OFF} is an instant, at which the "flipping" motion of the spacecraft was stopped, or, in other words, "switched OFF".

5.2. Study Case-3: "Switching OFF" flipping motion of the spacecraft after one flip (solution-2)

It is interesting to observe that stabilisation of the system, illustrated with Figs. 10 has been achieved with a controllable change of the moment of inertia I_{yy} (associated with the main rotation of the spacecraft), which initially had its value of ${}_iI_{yy} = 0.35$, being an intermediate value, surrounded by the smallest $I_{xx} = 0.2$ and largest $I_{zz} = 0.4$ moments of inertia:

$$I_{xx} < {}_iI_{yy} < I_{zz} \tag{17}$$

While keeping values of I_{xx} and I_{zz} unchanged, the value of I_{yy} in the presented experiment was changed from ${}_iI_{yy} = 0.35$ to the final value of ${}_fI_{yy} = 0.5$, as per Fig. 8, after which it became the largest principal moment of inertia:

$$I_{xx} < I_{zz} < {}_fI_{yy} \tag{18}$$

Figure 10 shows that as one of the consequences of the increase of I_{yy}, was a reduction from 15 to 10.5 rad/s of the associated angular velocity ω_y of the spacecraft. This simulation result is in perfect agreement with the conservation of the angular momentum of the system, suggesting that the rotational speed must be reduced by the ratio of $15 \times ({}_iI_{yy}/{}_fI_{yy}) = 15 \times (0.35/0.5) = 10.5$ rad/s.

In contrast to Case-1, where ω and H plots had similar shapes, in the Case-3 these plots are different. Figure 11(b) shows that morphing did not change the angular momentum H_y and after the morphing was completed, the value of H_y stayed almost unchanged, evidencing that the stopping "flipping" motion has been successful.

5.3. Study Case-4: "Switching OFF" flipping motion of the spacecraft after two flips (solution-1)

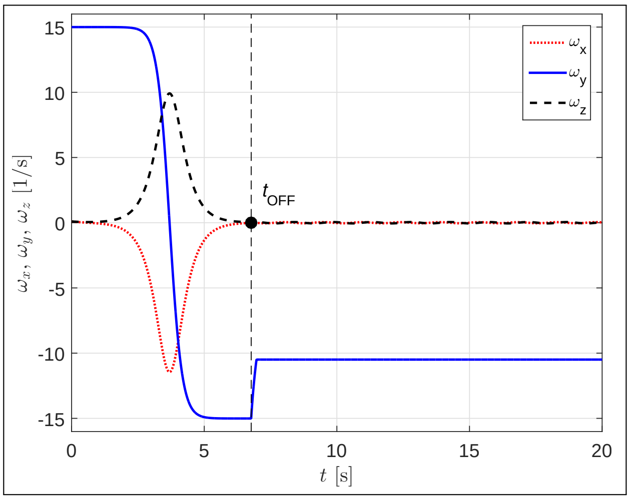
We now demonstrate switching OFF the "flipping" motion of the morphing spacecraft after it performs two flips. The time history of morphing is similar to presented in Fig. 8, but morphing is starting at $t = 13.54$ s. Results of this Case-4 are presented in Fig. 12. Observed reduction of the angular velocity ω_y is the same, as for the Case-2, however, in Case-2 after the motion is stabilised, the spacecraft continues its flight backwards, whereas in the current case, the stabilised attitude of the spacecraft is the same as at the initial time.

5.4. Study Case-5: "Switching ON" spacecraft flipping motion

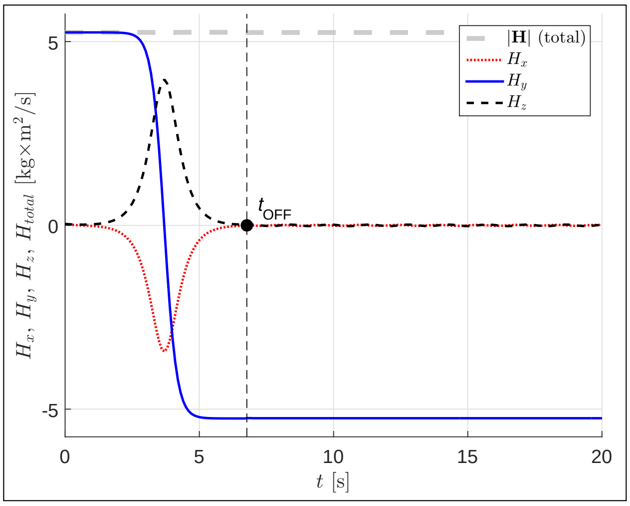
In a similar way as stabilisation, described in the Cases 2-4 was achieved, we can initiate the "flipping" motion of the spacecraft. For this, the axis of the major rotation of the system (let say, y) initially should coincide with the axis of minimal or maximal moments of inertia, i.e. one of the conditions should be satisfied: $I_{yy} < \min(I_{xx}, I_{zz})$ or $I_{yy} > \max(I_{xx}, I_{zz})$. In this case initiated rotation would be stable, without "flipping". To activate the "flipping" motion, morphing of the system should be performed, which should result in I_{yy} becoming an intermediate value between I_{xx} and I_{zz}. In the study Case-5, as illustration, we use the following values: $\omega_x = 0.1$, $\omega_y = 26.25$, $\omega_z = 0.1$ (all - in rad/s), $I_{xx} = 0.3$, ${}_iI_{yy} = 0.2$, ${}_fI_{yy} = 0.35$, $I_{zz} = 0.4$ (all - in kg×m^2). The time history of applied morphing is presented in Fig. 13 and the results of the simulation are shown in Fig. 14.

5.5. Study Case-6: "Switching ON" spacecraft flipping motion with following "Switching OFF"

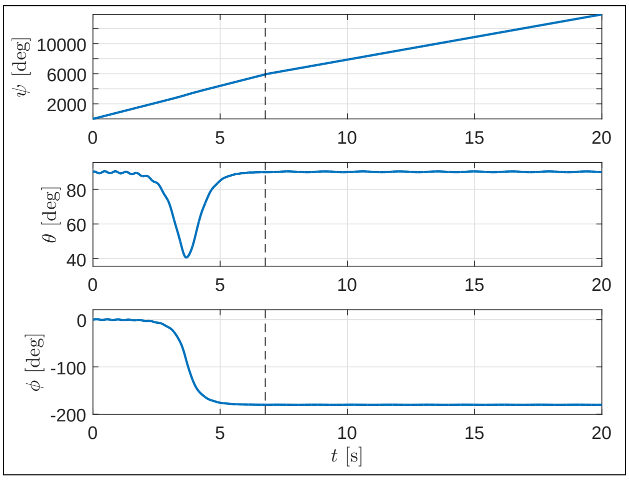
Case-6 represents further development of the Case-5 by switching OFF the "flipping" motion at $t = 9.89$ s, instant of



(a) Time histories of the angular velocity components of the spacecraft.



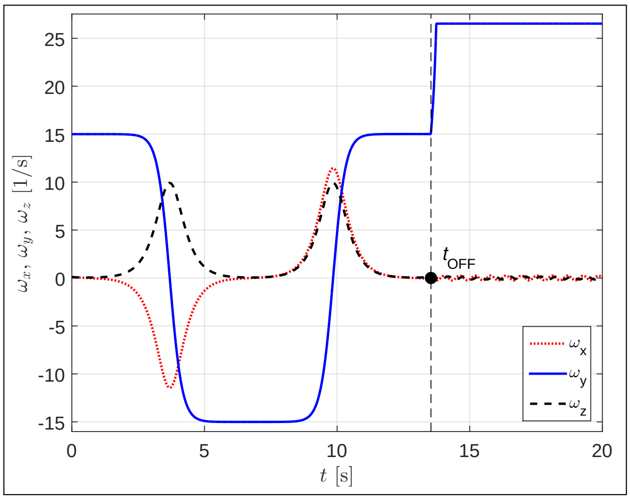
(b) Time histories of the angular momenta of the spacecraft.



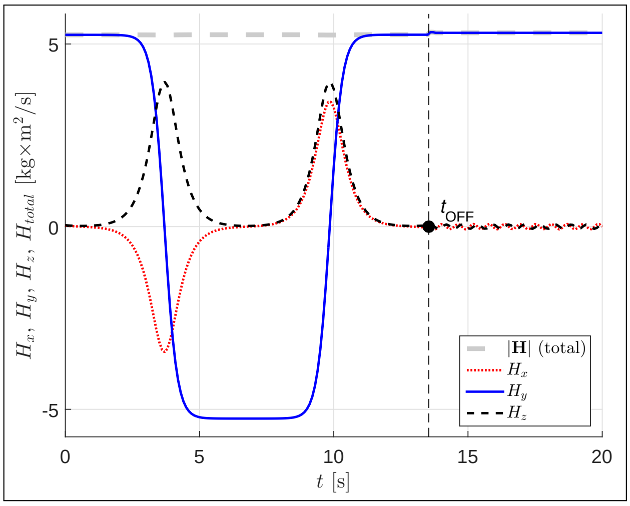
(c) Time histories of the angular momenta of the spacecraft.

Fig. 11. Simulation results for the study Case-3: switching off the ”flipping” motion of the spacecraft.

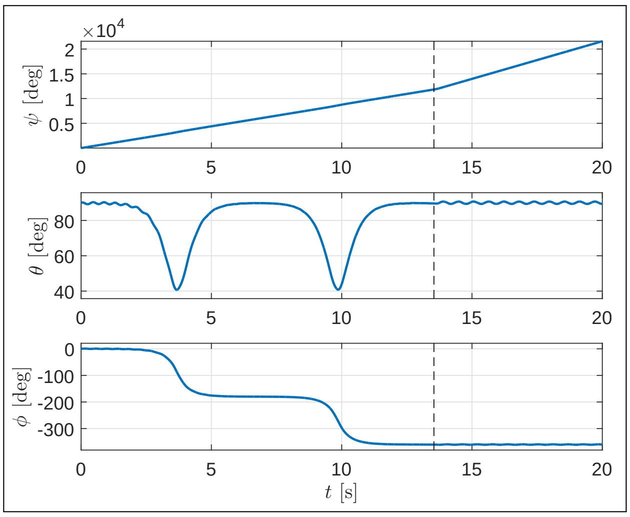
the maximal value of ω_y. The time history of applied morphing is presented in Fig. 15 and the results of the simulation are shown in Fig. 16.



(a) Time histories of the angular velocity components of the spacecraft.



(b) Time histories of the angular momenta of the spacecraft.

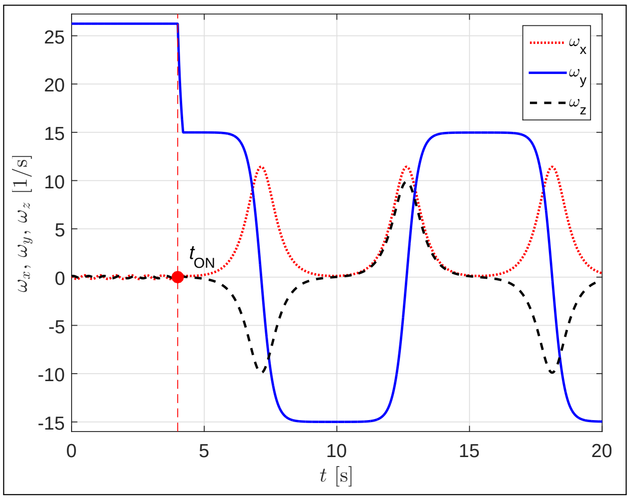


(c) Time histories of the angular momenta of the spacecraft.

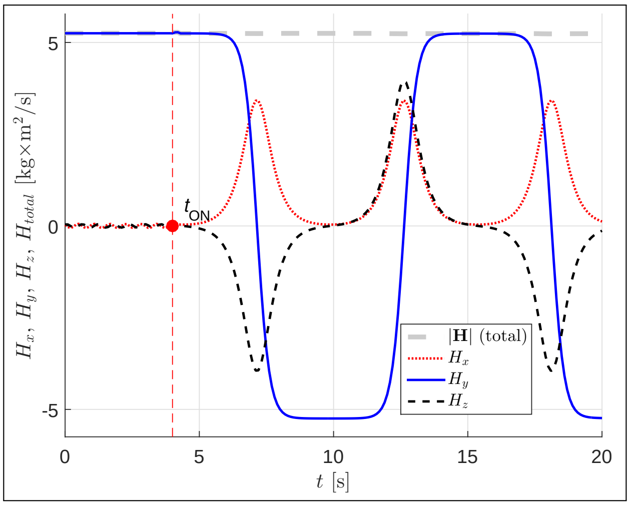
Fig. 12. Simulation results for the study Case-4: switching off the ”flipping” motion of the spacecraft.

6. Main Results and Conclusions

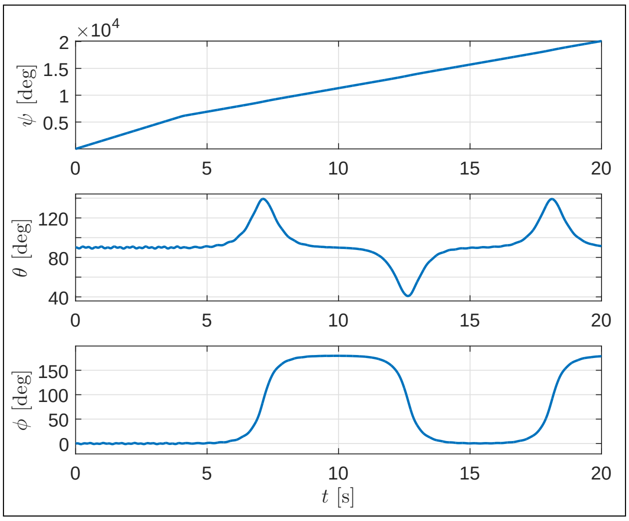
This paper is dedicated to the numerical simulation and analysis of the “Dzhanibekov’s Effect” - non-stable “flipping” motion of the rigid body with periodic change by 180° of the di-



(a) Time histories of the angular velocity components of the spacecraft.



(b) Time histories of the angular momenta of the spacecraft.



(c) Time histories of the angular momenta of the spacecraft.

Fig. 14. Simulation results for the study Case-5: activation of the ”flipping” motion of the spacecraft.

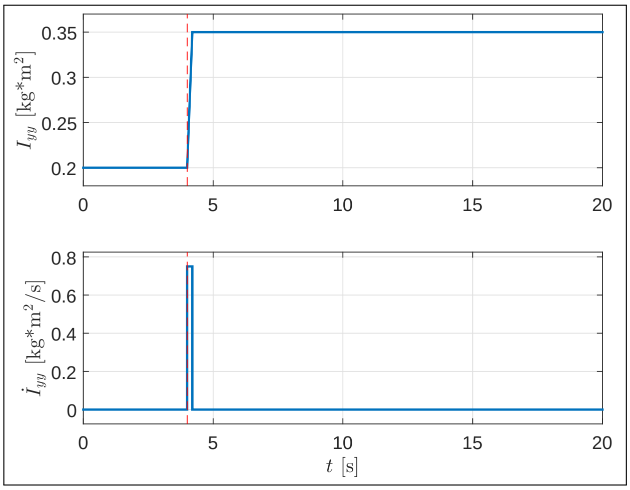


Fig. 13. Time history of the controlled manipulation with the moment of inertia I_{yy} to stop flipping motion of the system.

rection of the main axis of its rotation, always occurring when the body is provided with the main rotation about its axis with intermediate principal moment of inertia.

In this work we proposed and developed a new concept of utilizing “Dhanibekov’s Effect” for changing attitude of the spacecraft via its inertial morphing, without employing classical gyroscopes. Moreover, we proposed and tested a new method of switching OFF the “flipping” motion on the main axis of rotation by transferring motion to the stable (i.e. “non-flipping”) mode. For the implementation of this transformation, we proposed two main conceptual solutions, involving changes to the system, resulting in the intermediate moment of inertia becoming the smallest or largest principal moment of inertia of the body. A conceptual model of the 6-mass model of the spacecraft enabling controllable switching OFF of the “Dzhanibekov’s Effect” flipping is presented.

Furthermore, implementation of the transfer of the stable motion of the spacecraft to the unstable (i.e. “switching ON” the “flipping” mode) has also been successfully completed.

These “flipping” mode “switching ON” and “switching OFF” capabilities and their combinations have been successfully demonstrated on the representative study cases.

To achieve the main objectives in the study, we first developed a model of the tumbling spacecraft, based on the nonlinear Euler’s equations of rigid body motion and successfully simulated classical “Dzhanibekov’s Effects” for the rigid bodies with constant inertia properties. As main further development we enhanced the analytical and numerical models, which

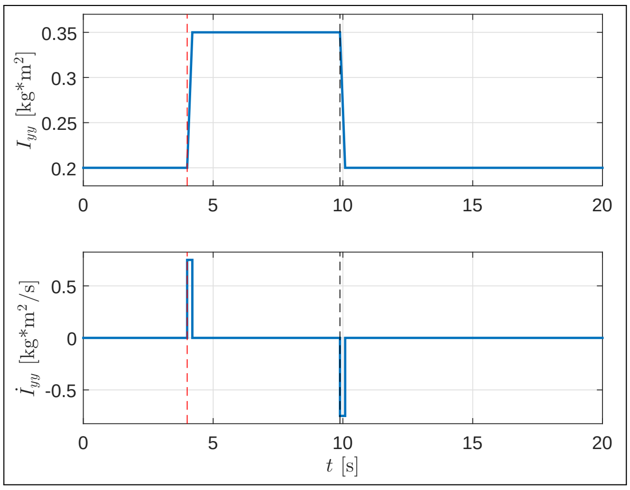
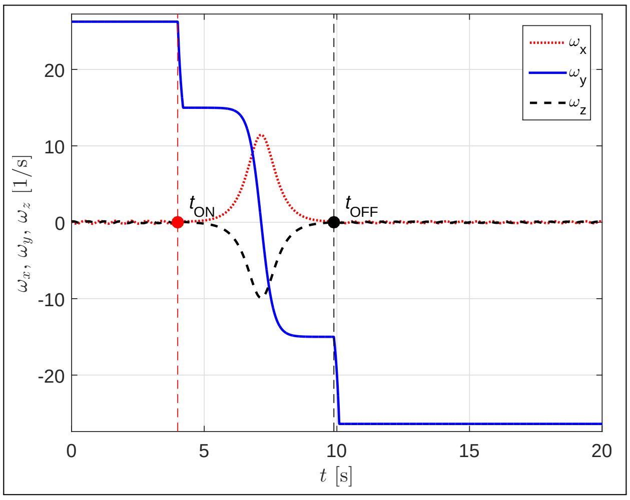
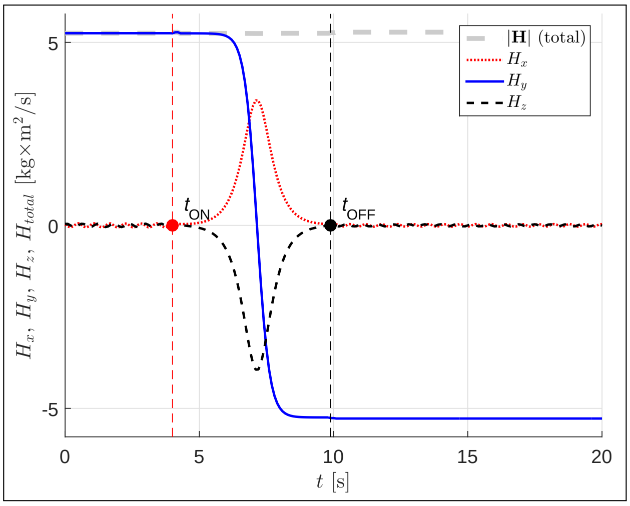


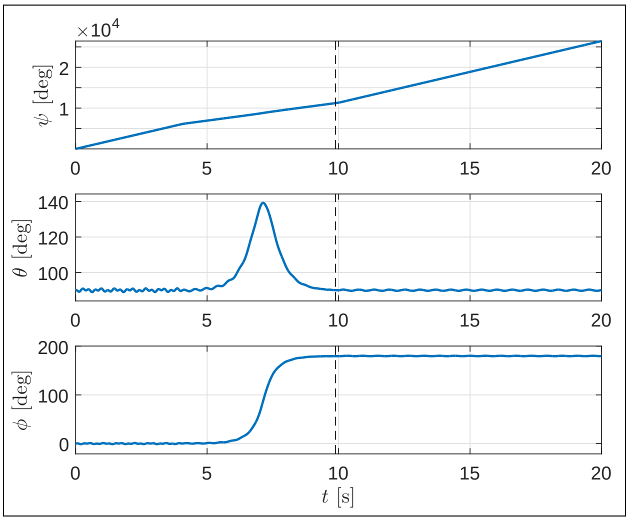
Fig. 15. Time history of the controlled manipulation with the moment of inertia I_{yy} to activate flipping motion of the system and then to turn it off.



(a) Time histories of the angular velocity components of the spacecraft.



(b) Time histories of the angular momenta of the spacecraft.



(c) Time histories of the angular momenta of the spacecraft.

Fig. 16. Simulation results for the study Case-6: activation of the "flipping" motion of the spacecraft.

enabled us to simulate wide class of systems with inertial morphing (i.e. systems with *variable* moments of inertia). It has been proven that the conservation of angular momentum in the morphing system is observed.

It has been demonstrated that time of activation of the spacecraft controllable morphing and its duration are critical factors in the quality of stabilisation and "flipping" de-stabilisation processes. Based on the simulated tests, we proposed recommendations on morphing of the spacecraft.

At last, a new graphical interpretation of the transfer of the proposed morphed spacecraft from stable mode to the "Dzhanibekov's Effect" "flipping" mode and vice versa, have been presented.

References

1) Komlev, G. Dzhanibekov's photo from USSR stamp, "Soyuz-27", 1979. `https://commons.wikimedia.org/wiki/File:USSR_stamp_Soyuz-27_1978_4k.jpg`, (accessed April 09, 2017).
2) Dzhanibekov, Vladimir Aleksandrovich. Wikipedia (In Russian). `https://ru.wikipedia.org/wiki/%D0%94%D0%B6%D0%B0%D0%BD%D0%B8%D0%B1%D0%B5%D0%BA%D0%BE%D0%B2,_%D0%92%D0%BB%D0%B0%D0%B4%D0%B8%D0%BC%D0%B8%D1%80_%D0%90%D0%BB%D0%B5%D0%BA%D1%81%D0%B0%D0%BD%D0%B4%D1%80%D0%BE%D0%B2%D0%B8%D1%87` (accessed April 12, 2017).
3) Djanibekov, V.A.: Interview at the "Secret Signs" TV Program, `https://youtu.be/dL6Pt1O_gSE` (published Feb 19, 2010, accessed April 12, 2017).
4) Roskosmos Telestudio: Dzhanibekov's Effect, `https://youtu.be/6ozUSgBjeaQ` (published on May 15, 2012; accessed April 12, 2017).
5) Murakami, H., Rios, O. and Impelluso, T.J.: A Theoretical and Numerical Study of the Dzhanibekov and Tennis Racket Phenomena, *Journal Applied Mechanics*, **83**, (Sep 08, 2016), No. 11, 111006 (10 pages). Paper No: JAM-16-1017, doi:10.1115/1.4034318.
6) Kawano, D.T.: A Tumbling T-Handle in Space, *Rose-Hulman Institute of Technology*. `http://rotations.berkeley.edu/?page_id=2472` (accessed April 12, 2017).
7) Roman Vladimirovich F.: Animation of the Dzhanibekov's Effect, `https://drive.google.com/drive/u/1/folders/0B3MWZcYwMLYoRkVOMXdsdDAxS0E` (accessed April 12, 2017).
8) Djanibekov, V.A.: Tumbling Earth: Interview at the "I Came Here to Fly" the First Educational TV Channel Program, `https://youtu.be/cy30_6m1jPc?t=901` (accessed April 09, 2017).
9) Shkaplerov, A. and Burbank, D.: Experiments on board of the International Space Station (Expedition #30, 2011). `https://youtu.be/LzVItPwiQyI` (published Feb 11, 2013; accessed April 12, 2017).
10) Wakata, K.: Mysteries of Rotational Movement (Onboard of the ISS, Expedition #38, 2014), JAXA. `https://youtu.be/QhSN2eua14I?t=309` (published Nov 25, 2013; accessed April 06, 2017).
11) Ford, K.: Working with the Spheres Satellites Onboard of the ISS (Expedition #34, 2013) `https://youtu.be/dsXOxcDSBLQ?t=108` (published Mar 13, 2013; accessed April 09, 2017).
12) Garriott, R.: Space Video Blog: Rotational Inertia, `https://youtu.be/fPI-rSwAQNg` (accessed April 12, 2017).
13) Plasma, B.:A T-handle tumbling in space aboard the International Space Station. `https://youtu.be/1n-HMSCDYtM` (published Mar 02, 2009; accessed April 12, 2017).
14) Portrait of L.Euler. University of Tartu collection, `http://dspace.ut.ee/handle/10062/22581?locale-attribute=en`, (accessed April 12, 2017).
15) Euler, L.: "Du mouvement de rotation des corps solides autour d'un axe variable," *Mémoires de l' académie des sciences de Berlin*, Vol. 14, Berlin Academy, Berlin, Germany, 1758, pp. 154193.
16) The Euler Archive, `http://eulerarchive.maa.org/` (accessed April 05, 2017).
17) Marquina, J.E., Marquina, M.L. , Marquina, V. and Hernández-Gómez, J.J.: Leonhard Euler and the mechanics of rigid bodies. *European Journal of Physics*, October 2016, **38**(2017), No.1, 015001 (11pp), doi:10.1088/0143-0807/38/1/015001.
18) Marsden, J. E. and Ratiu, T. S.: Introduction to Mechanics and Symmetry. A Basic Exposition of Classical Mechanical Systems. Springer, 1999. - 693pp. (ISBN-13: 978-0387986432, ISBN-10: 038798643X).